My method is DP (top-down + memorization). First consider how to get the result f(n) from previous result f(0)...f(n-1).  
Actually, the result f(n) will be put an extra () pair to f(n-1). Let the "(" always at the first position, to produce a valid result, we can only put ")" in a way that there will be i pairs () inside the extra () and n - 1 - i pairs () outside the extra pair.

Let us consider an example to get clear view:

f(0): ""

f(1): "("f(0)")"

f(2): "("f(0)")"f(1), "("f(1)")"

f(3): "("f(0)")"f(2), "("f(1)")"f(1), "("f(2)")"

So f(n) = "("f(0)")"f(n-1) , "("f(1)")"f(n-2) "("f(2)")"f(n-3) ... "("f(i)")"f(n-1-i) ... "(f(n-1)")"